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# MIGRATION, FERTILITY, AND AGING IN STABLE POPULATIONS\*

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*Fertility is below replacement level in all European countries, and population growth is expected to decline in the coming decades. Increasing life expectancy will accentuate concomitant aging of the population. Migration has been seen as a possible means to decelerate aging. In this article, I introduce a stable, open-population model in which cohort net migration is proportional to births. In this case, the migration-fertility trade-off can be studied with particular ease. I show that although migration can increase the growth rate, which tends to make the age distribution younger, it also has an opposite effect because of its typical age pattern. I capture the effect of the age pattern of net migration in a migration-survivor function. The effect of net migration on growth is quantified with data from 17 European countries. I show that some countries already have a level of migration that will lead to stationarity. For other countries with asymptotically declining population, migration still provides opportunities for slowing down aging of the population as a whole.*

**T**he aging and forecasted decline of populations of industrialized countries have lead to attempts to discover the extent to which increasing migration could alter the trends (e.g., United Nations, Population Division 2000). One difficulty in making such calculations is that one can make numerous alternative assumptions concerning how migration occurs. It may not always be easy to check the extent to which results depend on the particular choices.

Despite its lack of realism, stable population theory<sup>1</sup> provides explicit formulas that can be used for insight (e.g., Espenshade, Bouvier, and Arthur 1982). However, even the stable model can be formulated in different ways, and alternative formulations provide different insights. The assumption of a constant stream of immigrants has been the most popular choice (e.g., Bacaër 2003; Mitra 1983, 1990; Schmertmann 1992; Schmidbauer and Rösch 1995; Wu and Li 2003). However, as discussed by Pollard (1966) and Liao (2001), for example, a proportionality assumption also makes sense. In fact, in a full multistate stable model, net migration would be proportional to the size of any age group (e.g., Rogers 1995:118).

In this article, I assume that (the positive or negative) cohort net migration is proportional to births. This will lead to a particularly simple analysis in which the trade-off between fertility and net migration can explicitly be studied, so the results complement those obtained earlier (e.g., Schmertmann 1992; Wu and Li 2003). Formulas for growth rates and age distribution are similar to those of the closed population case.

The second section of this article defines the proportional version of the stable population model and illustrates the key migration-survivor function with data from the Nordic countries. Mathematical arguments will be heuristic throughout; to avoid a disruption in

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1. By stable population, I mean a population that grows or declines exponentially and whose vital rates and age distribution do not change (Keyfitz 1977:51). This is unrealistic. In particular, death rates have tended to fall, thus increasing life expectancy and also accentuating the aging of a population. Nonetheless, stable population calculations can be informative. They reveal the long-term implications of different demographic assumptions, and they can be compared with calculations based on changes in vital rates.

continuity, some specifics are given in footnotes. The subsequent section indicates the effect of net migration on the growth rate. Numerical estimates are given for 17 European countries. Next, I discuss the fertility-migration trade-off by showing what levels of net migration and fertility lead to a stationary population and give empirical estimates based on data from the Nordic countries. Similarly, from dependency ratio data from the Nordic countries, I then consider the effect of migration on aging. I conclude by noting some implications of the stable population analysis.

## STABLE POPULATION WITH NET MIGRATION PROPORTIONAL TO BIRTHS

### Births and Migration

Suppose that the density of births at time  $t$  is  $b(t) = be^{\rho t}$ , where  $\rho$  is the (positive or negative) growth rate and  $b > 0$  is the density at  $t = 0$ . I assume that  $p(x)$  is the probability of surviving to age  $0 < x \leq \omega$ . Here,  $\omega$  is the highest possible age. Suppose that  $T > 0$  is the total fertility rate,  $\alpha > 0$  is the lowest age of childbearing, and  $\beta$  is the highest age of childbearing with  $\alpha < \beta < \omega$ . I consider the female population only, so  $T$  is numerically equal to the usual gross reproduction rate (cf. Shryock and Siegel 1976:314). The age-specific fertility in age  $\alpha \leq x \leq \beta$  is of the form  $Tf(x)$ , where  $f(x) \geq 0$  integrates to 1 over childbearing ages. I assume that the same survival and fertility values apply to all members of the population of interest, as long as they stay in it.

Define  $N(x, y)$  as the cumulative net migration to the population of interest, by members of those born at time  $y$ , by age  $0 < x \leq \omega$ . Its density is  $(d/dx)N(x, y) = n(x, y)$ . I shorten  $N(\omega, y) = N(y)$ . I assume that the age pattern of net migration does not change over time, so I write  $n(x, y) = n(x)N(y)$ ,<sup>2</sup> where

$$\int_0^{\omega} n(x) dx = 1. \quad (1)$$

### Migration Proportional to Births

The proportionality assumption that leads to stability is that I take  $N(y) = cb(y)$  for some  $c \neq 0$ . Alternative values of  $c$  lead to alternative stable populations. It follows that  $n(x, y) = cn(x)b(y)$ . For notational simplicity, I define the function

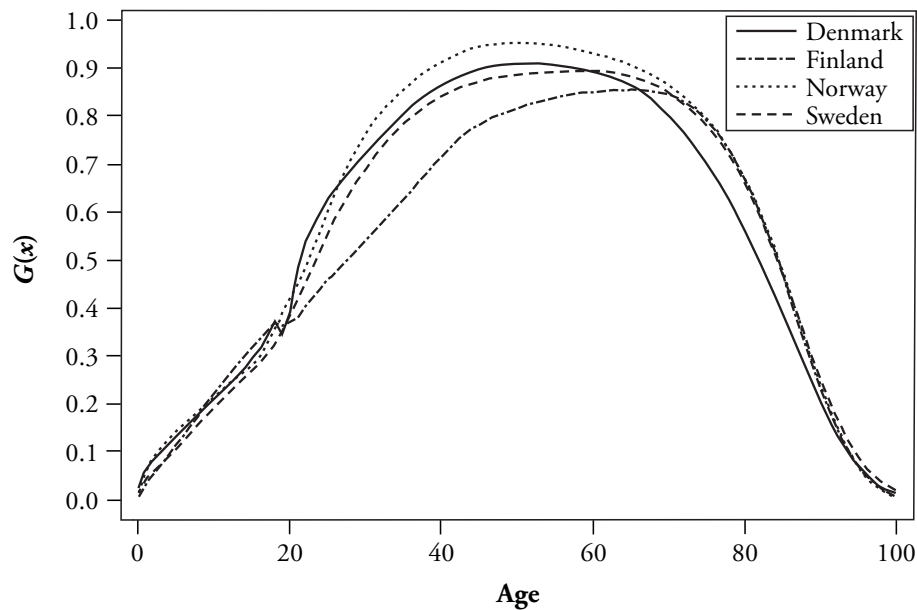
$$G(x) = \int_0^x n(z) \frac{p(x)}{p(z)} dz. \quad (2)$$

This concept was introduced by Espenshade et al. (1982:128) for immigration, but the formalism is applicable in the case of net migration also. Thus, restricting attention to those born at  $y$  who are in age  $x$  at time  $y + x$ , note that  $p(x)$  equals the number surviving from birth to age  $x$ , per original cohort member. Similarly,  $cG(x)$  is the number of  $x$ -year-old survivors added to the cohort by net migration, per original cohort member.<sup>3</sup> Consider now the cohort that is age  $x$  at time  $t$ . Using Eq. (2), I can then write its population density as  $b(t - x)\pi(x, c)$ , where I have shortened  $\pi(x, c) = p(x) + cG(x)$ .

Both  $n(x)$  and  $G(x)$  are independent of the level  $N(y)$  of net migration. Only the age pattern matters. For lack of a better term, I call  $G(x)$  a *migration survivor function*. For intuition, Figure 1 shows the graph of  $G(x)$  for four Nordic countries with good-quality

2. I assume that  $N(y) \neq 0$ . Using more complex notation, this restriction could be avoided. As a density of a "signed measure,"  $n(x)$  can take both positive and negative values (cf. Friedman 1982:25).

3. There are no "net migrants," but  $G(x)$  keeps track of the fraction surviving to age  $x$  of those who enter at any age  $z$  and stay, and subtracts the fraction of those leaving at  $z$ , who would survive if they stayed. I assume that survival in the population of interest is independent of the propensity to leave.

**Figure 1.** Migration Survivor Function  $G(x)$  for Denmark, Finland, Norway, and Sweden

migration data, in 2003.<sup>4</sup> I find a somewhat irregular rise reflecting the particular age pattern of the country's net migration in young adult ages, and then a more regular decline determined by mortality. In particular, the age distribution of migrants is quite different from that of the host population. One implication of the nonmonotonicity of  $G(x)$  is that stable populations that are open to migration do not have to have a monotone declining age distribution. This has been observed earlier (e.g., Mitra 1990) for the steady immigration populations.

### Population Renewal

From the argument of the previous section, it follows that earlier births generate the births at  $t$  via the basic renewal equation,

$$b(t) = T \int_{\alpha}^{\beta} b(t-x) \pi(x, c) f(x) dx. \quad (3)$$

When  $c = 0$ , I have the usual closed population renewal equation. Substituting the exponential form of the births into Eq. (3) results in the following equation:

$$1 = T \int_{\alpha}^{\beta} e^{-\rho x} \pi(x, c) f(x) dx. \quad (4)$$

4. My data are estimates of the so-called UPE Project for the year 2003. For some countries (e.g., the Nordic countries), data were available for 2003; for other countries, the values are one-year-ahead forecasts. In both cases, the values were smoothed over age and time, so they represent approximately average net migration after 2000. The values used are available at [www.stat.fi/tup/euupe/](http://www.stat.fi/tup/euupe/).

This connects the three parameters:  $\rho$ ,  $c$ , and  $T$ . In analogy with the closed population, the stable population experiencing proportional net migration has an age distribution proportional to  $e^{-\rho x}\pi(x, c)$ .<sup>5</sup>

### EFFECT OF MIGRATION ON GROWTH RATE

In a multistate system, the proportionality factor  $c$  would be determined as a part of the stable population calculation. Without a full model, this is not available, and I will simply replace the theoretical cohort rate by the empirically observed ratio of current (period) net migration and current births.<sup>6</sup> The column for  $c$  in Table 1 contains values obtained in this manner from 17 European countries in 2003. I caution that for many of the countries, reliable estimates of net migration do not exist (e.g., Eurostat 2004; Poulain 1993). Thus, for countries with poor migration data, the estimates of  $c$  in Table 1 are illustrative only. The column for  $\rho^*$  contains the conventional intrinsic growth rate (or intrinsic rate of natural increase; cf. Keyfitz 1977:177) that is obtained from Eq. (4) by taking  $c = 0$ ,  $\alpha = 15$ , and  $\beta = 50$ . The stable growth rate is the solution for  $\rho = \rho(c, T)$ , which is obtained using the values of  $c$  and  $T$  from the first two columns.

The column for  $T$  shows that all countries are far below replacement level.<sup>7</sup> The column for  $c$  shows that all countries are in-migration countries. Less obviously, countries with lower than average fertility have higher than average net migration. Notably, according to the available data, net migration in Spain exceeds births by 37%. In fact, the correlation between  $T$  and  $c$  is  $-0.64$  (with an approximate  $p$  value of .004 for the hypothesis of zero correlation). In other words, low fertility is associated with high net migration, and vice versa. Note also that the dispersion across countries (as measured by standard deviation) is much larger in terms of migration,  $c$ , than in terms of fertility,  $T$ .

Intrinsic growth is negative for all countries. With the exception of Ireland and France, the values indicate a decline in population size of 0.6% to 1.7% per year. The average rate of decline is 0.93%. However, net migration alleviates the effect considerably, so the average stable rate of decline is 0.25%, with five countries showing positive growth. At the same time, net migration decreases the dispersion in the rate of growth across countries from 0.5% to 0.4%. Although concurrent correlation does not measure causality, the findings are consonant with the hypothesis that countries regulate growth via migration.

### Trade-off Between Fertility and Migration

For any value of  $\rho$ , Eq. (4) can be considered as defining a relation between  $T$  and  $c$ . In fact, defining

$$\begin{aligned} A &= \int_{\alpha}^{\beta} e^{-\rho x} p(x) f(x) dx, \\ B &= \int_{\alpha}^{\beta} e^{-\rho x} G(x) f(x) dx, \end{aligned} \quad (5)$$

I can write Eq. (4) as  $1 = T(A + cB)$ , so  $T = 1 / (A + cB)$  and  $c = (1 - TA) / TB$ . Thus, the relationship is one to one.<sup>8</sup> Taking  $\rho = 0$ , I have the special case of a stationary population.

5. Because the population cannot become negative, one must have  $\pi(x, c) \geq 0$ . Therefore, the constant  $c$  must satisfy  $p(x) \geq -cG(x)$  for  $0 \leq x \leq \omega$ .

6. As pointed out by a referee, this estimate is valid in a stable model population with  $\rho = 0$ . The general case is more complex, but if  $n(x) \geq 0$  and  $\rho > 0$ , it underestimates  $c$ ; and if  $\rho < 0$ , it overestimates  $c$ .

7. A two-sex total fertility rate of approximately 2.07 is required for the countries listed for renewal. Taking 1.05 as the sex ratio at birth, I deduce that a gross reproduction rate of  $2.07 / 2.05 \approx 1.01$  is required for stationarity.

8. One must have  $T > 0$ , and  $c$  must satisfy the condition described in footnote 5.

**Table 1.** Stable Population Parameters in Selected European Countries: Female Total Fertility Rate ( $T$ ), Ratio of Net Migration to Births ( $c$ ), Intrinsic Growth Rate as a Percentage ( $\rho^*$ ), and Stable Growth Rate as a Percentage ( $\rho$ )

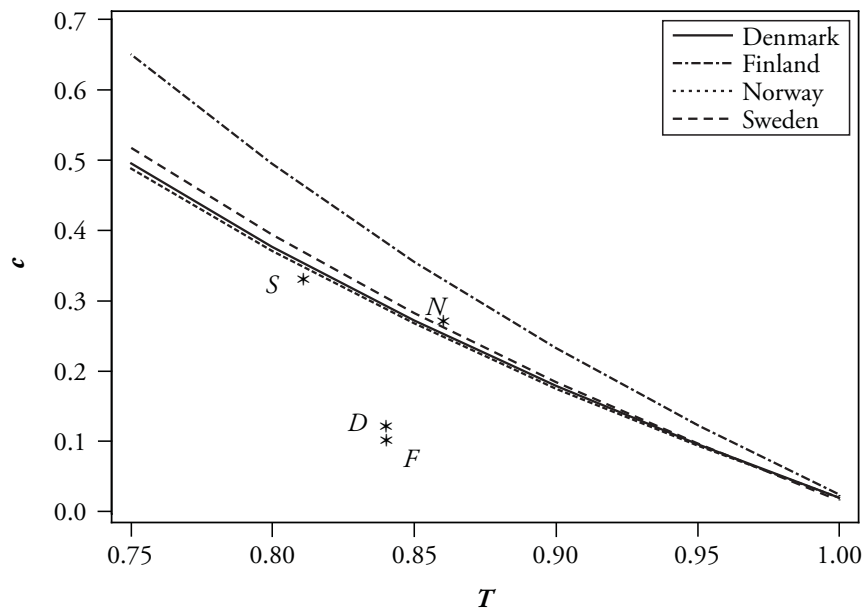
Country	$T$	$c$	$\rho^*$ (%)	$\rho$ (%)
Austria	0.68	0.45	-1.36	-0.35
Belgium	0.79	0.37	-0.84	0.00
Denmark	0.84	0.12	-0.62	-0.34
Finland	0.84	0.10	-0.63	-0.45
France	0.92	0.13	-0.34	-0.10
Germany	0.64	0.21	-1.58	-1.15
Greece	0.61	0.38	-1.66	-0.82
Ireland	0.95	0.48	-0.20	0.25
Italy	0.62	0.96	-1.59	-0.38
Luxembourg	0.80	0.47	-0.81	0.21
Netherlands	0.84	0.02	-0.60	-0.56
Norway	0.86	0.27	-0.56	0.03
Portugal	0.72	0.54	-1.19	-0.38
Spain	0.61	1.37	-1.62	0.16
Sweden	0.81	0.33	-0.75	-0.09
Switzerland	0.68	0.63	-1.32	0.11
United Kingdom	0.80	0.18	-0.81	-0.39
Average	0.78	0.39	-0.93	-0.25
<i>SD</i>	0.11	0.34	0.49	0.37

Table 1 indicates that the current level of migration already leads to positive growth in some countries, but it is of some interest to investigate the trade-off more closely.

Figure 2 displays pairs  $(T, c)$  that would maintain a stationary population in the four Nordic countries. As noted earlier in this article, I expect approximate stationarity at  $T = 1$  in the absence of migration. However, Figure 2 also shows that if fertility in, say, Norway declines to  $T = 0.75$ , then (maintaining current survival probabilities, distribution of fertility, and distribution of net migration) net migration would have to be approximately one-half of the births to maintain stationarity. In the figure, the current  $(T, c)$  values from Table 1 are individually marked: Denmark is at  $D = (0.84, 0.12)$ , Finland is at  $F = (0.84, 0.10)$ , Norway is at  $N = (0.86, 0.27)$ , and Sweden is at  $S = (0.81, 0.33)$ . Denmark and Finland are clearly below their respective curves. This is in accordance with Table 1, which shows that with current levels of fertility and net migration, these countries have negative growth. Sweden is almost on the curve (its current stable growth rate is essentially zero), and Norway is slightly above it.

These comparisons suggests that migration can have a substantial effect on population growth in the long run. The details across the countries differ, however. For example, to achieve stationarity, the level of net migration would have to be  $c = 0.29$  in Denmark, but Finland would have to reach  $c = 0.38$ . This difference is primarily attributable to the “younger”  $G(x)$  function of Denmark.

**Figure 2.** The Trade-off Between Fertility and Net Migration: Pairs of Values ( $T$ ,  $c$ ) That Maintain a Stationary Population for Denmark, Finland, Norway, and Sweden



Note: Actual values are marked with  $D$  for Denmark,  $F$  for Finland,  $N$  for Norway, and  $S$  for Sweden.

## AGING VIA DEPENDENCY RATIOS

### Dependency Ratios

In a closed, stable population, age distribution is proportional to  $e^{-\rho x}p(x)$  for  $0 \leq x \leq \omega$ , so a growing population with  $\rho > 0$  is younger than a declining population with  $\rho < 0$ . However, if  $\rho$  is fixed in an open population, there is a trade-off between  $c$  and  $T$ . Thus, a second aging effect can potentially derive from the relative values of  $c$  and  $T$ .

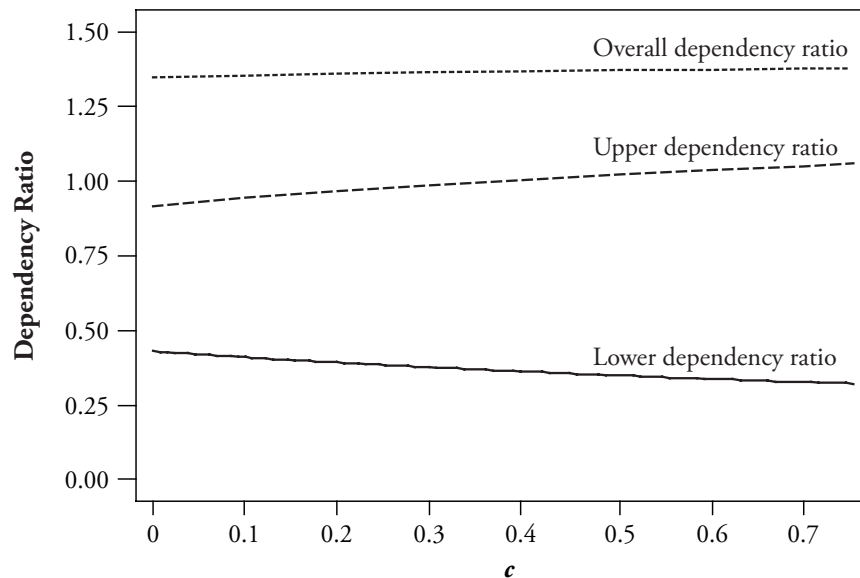
Age-dependency ratios are most often motivated by economic considerations, thus defining ratios for age groups 0–18, 19–64, and 65+, for example. However, because I concentrate on reproduction, I instead consider the ages 0 to  $\alpha$ ;  $\alpha$  to  $\beta$ ; and  $\beta+$ . Thus, I define

$$\begin{aligned}
 I_1(c, \rho) &= \int_0^{\alpha} e^{-\rho x} \pi(x, c) dx, \\
 I_2(c, \rho) &= \int_{\alpha}^{\beta} e^{-\rho x} \pi(x, c) dx, \\
 I_3(c, \rho) &= \int_{\beta}^{\infty} e^{-\rho x} \pi(x, c) dx.
 \end{aligned} \tag{6}$$

When multiplied by  $be^{\rho t}$ , these give the population at time  $t$  that is in pre-childbearing ages, in childbearing ages, and in post-childbearing ages, respectively. Then, I define the lower dependency ratio  $L(c, \rho) = I_1(c, \rho) / I_2(c, \rho)$ ; the upper dependency ratio  $U(c, \rho) = I_3(c, \rho) / I_2(c, \rho)$ ; and the overall dependency ratio  $D(c, \rho) = L(c, \rho) + U(c, \rho)$ . Using these



Figure 3. Average Across Countries of Lower Dependency Ratio, Upper Dependency Ratio, and Overall Dependency Ratio, Under the Assumption of Stationarity, in Denmark, Finland, Norway, and Sweden, as a Function of  $c$



measures, I conclude that net migration induces “aging” if  $L(c, \rho)$  decreases or  $U(c, \rho)$  increases with  $c$ .

### Empirical Estimates

To illustrate the possible second aging effect, I fix  $\rho = 0$ . In this case, the population is proportional to  $\pi(x, c)$  for  $0 \leq x \leq \omega$ . Figure 3 has a plot of  $L(c, 0)$ ,  $U(c, 0)$ , and  $D(c, 0)$  averaged across Denmark, Finland, Norway, and Sweden. From Figure 2, one might expect that an increase in net migration (and a concomitant decrease in fertility so that zero growth is maintained) would lead to a decrease in  $L(c, 0)$ . As shown in Figure 3, this is indeed the case. Numerically, the decrease is from  $L(c, 0) = 0.43$  at  $c = 0$  to  $L(c, 0) = 0.32$  at  $c = 0.75$ . Similarly,  $U(c, 0)$  increases from 0.92 to 1.06. The net effect is that  $D(c, 0)$  increases thus from 1.35 to 1.38. Although the overall increase is small, the analysis suggests that in these European countries, increased net migration is positively associated with aging when one controls for the rate of growth.

To see what the net effect on aging might be, consider Denmark and Finland, which have equal total fertility rates and negative stable growth rates of  $-0.34\%$  and  $-0.45\%$ , respectively. Assuming these rates of growth and the level of migration of Table 1, the line denoted as Current in Table 2 gives the corresponding dependency ratios. For example, the element  $L$  for Denmark is  $L(0.12, -0.0034) = 0.29$ . Suppose then that these countries maintain their current level of fertility of 0.84 but boost their net migration to the level leading to stationarity. From the data underlying Figure 2, I find that Denmark should have  $c = 0.29$  for Denmark, and Finland should have  $c = 0.38$ . Thus, net migration would have to be  $0.29 / 0.12 = 2.4$  times as high in Denmark and  $0.38 / 0.10 = 3.8$  times as high in Finland as in 2003. The resulting dependency ratios are on the line denoted as

**Table 2.** Age Dependency Ratios in Denmark and Finland, Under Current Conditions and Under Increased Migration That Leads to Stationarity

	Denmark					Finland				
	$\rho$ (%)	$c$	$L$	$H$	$D$	$\rho$ (%)	$c$	$L$	$H$	$D$
Current	-0.34	0.12	0.29	1.12	1.40	-0.45	0.10	0.30	1.27	1.57
Stationarity	0.00	0.29	0.38	0.93	1.31	0.00	0.38	0.38	1.04	1.41

Stationary in Table 2. For both countries, increasing net migration to reach stationarity would slow down population aging and help alleviate the burden on those members of working/reproductive ages.<sup>9</sup>

### Practical Magnitude of the Effect

One can contrast the effects noted herein to the effects that derive from increases in longevity. For several years, Finland has had the lowest level of net migration in the countries considered.<sup>10</sup> Using Finnish life tables compiled by Kannisto and Nieminen (1996) for five-year periods, I analyze the situation as follows. During 1950–1990, female life expectancy at birth increased from 69.9 to 78.8. Using the corresponding stationary populations, I find an increase of  $U(0,0)$  from 0.69 to 0.86. The data I use give the value 0.96 for Finland during 2003, when life expectancy at birth was 81.8 years. Thus, the increase in longevity by  $81.8 - 69.9 = 11.9$  years in 50 years' time (or 2.4 years in a decade),<sup>11</sup> is accompanied by an increase in the upper dependency ratio by  $0.96 - 0.69 = 0.27$  units. On the other hand, one can see from Table 2 that under the stable growth rate of  $\rho = -0.0045$ , another  $1.27 - 0.96 = 0.31$  units would be added; however, by increasing net migration to reach stationarity, the added amount could be reduced to  $1.04 - 0.96 = 0.08$  units. Thus, by boosting net migration to the level of zero growth, Finland could reduce the aging of its population by almost the same amount that mortality in the past half-century has added to it.

### CONCLUDING REMARKS

In this article, I assessed the migration-fertility trade-off using a simple, stable population model. This assessment was motivated by the fact that calculations under stability address directly the question of sustainability, or what combinations of fertility and net migration could coexist indefinitely.

I assumed an open population in which net migration is proportional to births. This provides an alternative to earlier analyses that assumed a steady inflow. In reality, there is always an outflow that is typically proportional to population size. In that case, a constant net flow assumption implies that the inflow is a linear function of the size of the receiving population. From this perspective, the two assumptions are not as far apart as they might first seem.

Both the level of net migration and the migration survivor function have identifiable effects on the age distribution. My findings partially support earlier studies that have been skeptical about the possibility of combating an aging population via increased net

9. These considerations can be extended in many ways: for example, the squared distance of any point  $(c^*, T^*)$  from the curve  $T = 1 / (A + cB)$  is  $H(c) = (1 / (A + cB) - T^*)^2 + (c - c^*)^2$ . Setting  $H(c) = 0$  leads to a polynomial equation in  $c$  that can be solved to find the closest point on the curve to  $(c^*, T^*)$ . Replacing the Euclidean distance by a weighted distance, I could incorporate the notion of an “optimal” path to a particular level of growth.

10. In Table 1, the Netherlands has an even lower value, but this reflected the conditions of 2003, and earlier net migration was higher.

11. This is essentially the rate of increase in the “best practice” life expectancy of Oeppen and Vaupel (2002).



migration. However, the effects are subtle. Migration can boost growth rate substantially. This tends to make the age distribution younger. However, if one controls for the growth rate, increasing migration at the expense of fertility tends to make the age distribution older because entering migrants are always older than the newly born.

Aging is, presumably, a desirable goal if it occurs via increases in longevity. However, it is less clearly desirable if it occurs via negative population growth. Although one can argue that from a broad ecological point of view, positive population growth can be non-sustainable, it is still possible that combating drastic population decline (by increasing net migration) increases welfare by easing the burden on the working-age population.

If there is room for growth, increasing net migration can markedly slow down the aging of a population. For a country with a low level of net migration, the effect can correspond in magnitude to aging caused by an increase of a decade or more in life expectancy. Note, however, that for most countries in this European data set, the effect would be less, and five of them have already “used up” the effect of growth rate; that is, their stable growth is positive. Should they wish to reduce net migration without accelerating aging, an increase in fertility is the only option.

I assumed that immigrants have the same vital rates as the native-born population. This seems reasonable because the heterogeneity induced by past migration is reflected in the current average vital rates that form the basis of the stable population analysis. Yet, if the possibly higher fertility of immigrants persists and their share increases, the aging effect of net migration would be smaller than estimated here. A more refined analysis is possible in a multistate context if vital rates are available by country of birth, for example. Similarly, this setting can be extended to assess the foreign-born proportion. This also requires data by place of birth (for  $c$ , as a minimum), and the details are somewhat more complex than those presented in this article.

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